Choose your Own Adventure: How much Power do I need?

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My Dilemma

- I have a set of problems that I need to solve before I am able to go out tonight.
- I want to solve them using the least powerful machine.
- Which will I choose first?

Randomly picking a shoe will I be guaranteed to find a match?

Do I have a three piece outfit I can wear?

Are the outfits worn by my friends and I all distinct?
Shoe Problem

● I want to know if I pick a shoe at random whether or not I will be guaranteed to find a match.

● How can I encode this into a language?
Language Encoding

- I figured that I could classify all of my shoes as either left or right.
- That could be simplified to a 0 or 1.
- If there was an equal number of them I would have a match for all of them.
- Therefore the language would be: $0^n1^n$
How do I solve this?

- I can not solve this using a DFA as proven by the Pumping Lemma but I can solve this with a PDA!
Outfit Problem

• I want to wear a three piece outfit consisting of a skirt, tank top, and shirt.
• All of my clothes in the closet are organized in outfits.
• Each piece has a number, ex. a skirt is a 1, tank top a 2, pants 3 etc.
• How can this be encoded as a language?
Language Encoding

- I know I have ten different articles of clothing in my closet and they are all in order in terms of outfits.
- So if I want an outfit of a skirt, tank top, and shirt I need to look for a sequence of those three numbers which would be a 124.
How do I solve this?

- Just simply looking for a particular sequence of numbers is easily solvable using a DFA!
Outfit Distinctness Problem

- I have five friends that I want to go out with tonight and none of us want to be wearing the same thing.
- How can we guarantee that we are all wearing something different?
Language Encoding

- We have determined that each of the outfits can be encoded using a string of numbers.
- This means that if we looked at each of the outfit encodings we should be able to tell if each one is distinct.
How do I solve this?

- If there are five people that I need to check against a DFA would not give me enough power.
- I also can’t use a PDA because I need multiple inputs.
- A multi-tape Turing machine would give me the power I need!
How do I solve this?

- Mark the first input with a special symbol
- Go through the elements of the first input checking against the second input
- Check the first input against all the inputs
- Once you have checked the first input against the others check the second input against the remaining inputs
- Continue this process until you are checking the last two inputs
- Accept if they are all distinct, otherwise reject
Are we done yet?

• Now that all those problems have been solved there is still one more problem to consider.
• Since we could do the problems in any order how many ways are there in which to view the slides if we are allowed to repeat?
• Lets try encoding it in a language.
Language Encoding

- We could number each of the slides but if we still have loops, we could go forever!
- There is no finite number of ways in which to do this.
- Does that mean this is an undecidable problem?
How do I solve this?

- In order to determine how tightly we can classify the language we have to see if it is countable or not.
- Is there a mapping of slide patterns to the natural numbers?
- Yes! We can look at all the configurations of the smallest length and continue from there.
- Thus there is an enumerator for the language but not a decider.
In summary

- We know have a list of problems and the ways in which they can be solved using varying degrees of power.
- Thus we can come up with a classification for each of them.