

Physics 204

Supplemental Problems for Assignment 2

Due Friday, 25 January 2008

1 Equilibrium of a Vertical Spring

An object is said to be at *equilibrium* if its linear momentum and angular momentum are constant. An object is at *static equilibrium* if its momentum and angular momentum are constant and zero (that is, it is not moving). In either case, the net force on the object must be zero, $\sum \vec{F} = 0$ or $\vec{F}_{net} = 0$.

In the horizontal case of a mass on a spring (where there is no friction between the mass and the surface on which it's moving) there will be no force exerted on the mass by the spring when it is at equilibrium and the spring is relaxed. It is therefore convenient to set up the coordinate system so that the origin $x = 0$ is at the equilibrium position and *Hooke's Law* takes the form $F_x = -kx$ in one dimension.

Newton's Second Law for this system yields the differential equation

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0, \quad (1)$$

a solution to which is $x(t) = x_{max} \cos(\omega_0 t)$ where $\omega_0^2 = k/m$ and we've ignored the phase constant.

If we take the same spring and hang the same mass from it in a vertical configuration, the length of the spring will be longer when it is in equilibrium than the relaxed length of the spring in the horizontal configuration. This is because the force of gravity on the mass displaces the equilibrium point of the system. However, if displaced from its new equilibrium, the mass on the spring will still oscillate about the new equilibrium position in the same way that it oscillated about the old equilibrium position. At least that's the claim. Now you get to show it.

- Draw a free body diagram for the mass hanging from the spring in the vertical configuration. Be sure to include all forces. Use the variable y for your coordinate system and take up to be positive.
- Apply *Newton's Second Law* to the mass on the spring.
- Rewrite the result from part (b) as a differential equation. Your result should be

$$\frac{d^2y}{dt^2} + \frac{k}{m}y + g = 0. \quad (2)$$

Compare this to equation 1.

- d) Now we want to find out *how much* gravity displaces the equilibrium. Note that $y = 0$ is **not** the equilibrium position of the system. Show that the new equilibrium position is $y_{eq} = -g\frac{m}{k}$ by allowing the acceleration (and therefore the sum of the forces in Newton's Second Law) to equal zero.
- e) Define a new vertical coordinate $z = y - y_{eq} = y + g\frac{m}{k}$. Solve this for y , substitute into equation 2, and simplify (be sure to take the second derivative of y with respect to time and substitute this in as well - note that $g\frac{m}{k}$ is a constant, so its derivative is zero). Compare this to equation 1. What value of z corresponds to the equilibrium position.

This process is what is known as a coordinate transformation in Physics. Coordinate transformations can greatly simplify the mathematics and allow us to describe more difficult problems by using easier problems, like the *Simple Harmonic Oscillator*.

2 Small Angle Approximation

Consider the small angle approximation discussed in class when going over the solution of $\theta(t)$ for the simple pendulum. We claimed that 15° is a good approximate upper limit for a small angle. To show why, make a table with four columns: θ (degrees), θ (radians), $\sin(\theta)$, and percent difference. Calculate percent difference between θ in radians and $\sin(\theta)$ using

$$\%difference = 100 \frac{|\sin(\theta) - \theta(\text{radians})|}{\sin(\theta)}.$$

Do these calculations for 0° to 45° in increments of 1° . Feel free to use a spreadsheet, but make sure it is nicely formatted as you learned to do in lab.

For what angle is the %difference about 1%? Why is 15° often cited as being a good approximate upper limit for a small angle?