Math 111 - Exam 1a

1) Evaluate the following limits:

a) $\lim\limits_{x\to\infty} \dfrac{1-4x^7}{1-3x^5}$ b) $\lim\limits_{x\to 6} \dfrac{x^3-36x}{x^2+x-42}$ c) $\lim\limits_{x\to 0} \dfrac{\tan(3x)}{x^2+x}$

d) $\lim\limits_{h\to 0} \dfrac{(1+h)^{100}-1}{h}$

2) Calculate the derivatives of the following. DON'T SIMPLIFY!

a) $y = \dfrac{\sin(x)-2\cos(x)}{\sqrt{3}}$ b) $y = \dfrac{2-\frac{1}{x}}{x+1}$

c) $f(x) = \sqrt[3]{x}\sec(x)$ d) $s = t^2\left(\sqrt{t} - 3t^{3/4} + \dfrac{1}{t^4}\right)$

3) Sketch the graph of a function f(x) satisfying all of the following. Be sure to label your axes.

a) $\lim\limits_{x\to -3} f(x) = \infty$ e) $f(0) = 2$ i) $f(1) = 0$

b) $\lim\limits_{x\to -2^-} f'(x) = 0$ f) $\lim\limits_{x\to 0} f(x) = 1$ j) $\lim\limits_{x\to\infty} f(x) = -\infty$

c) $\lim\limits_{x\to -2^+} f'(x) = 1$ g) $\lim\limits_{x\to 1^-} f(x) = 2$

d) $f'(-1) = 0$ h) $\lim\limits_{x\to 1^+} f(x) = 0$

4) a) Find $f'(x)$, using the limit definition of derivative, for $f(x) = \dfrac{x}{x+1}$

b) Find the points on the curve $y = 2x^2 - 3x + 8$ at which the tangent line goes through the origin.

Math 111 - Exam 1b

1. Find the following limits if they exist. If they don't exist describe the behavior of the function near the point of interest. Indicate justification for each answer.

a) $\lim\limits_{x \to -1} \dfrac{x^2 + 3x + 2}{x^2 + x}$ b) $\lim\limits_{x \to 2} \dfrac{x+1}{x-2}$

2. What does it mean to say a function f is continuous at x = a?

3. For a function g what is the definition of g´(x)?

4. Use the definition of derivative to find f´(x) for f(x) = $\dfrac{1}{x}$.

5. Describe the following two types of discontinuity. If you want to use pictures or an example, you may.
   a) jump discontinuity
   b) removable discontinuity

6. Use the δ - ε definition of limit to verify that $\lim\limits_{x \to 2} \dfrac{x^2 - x - 2}{x - 2} = 3$.

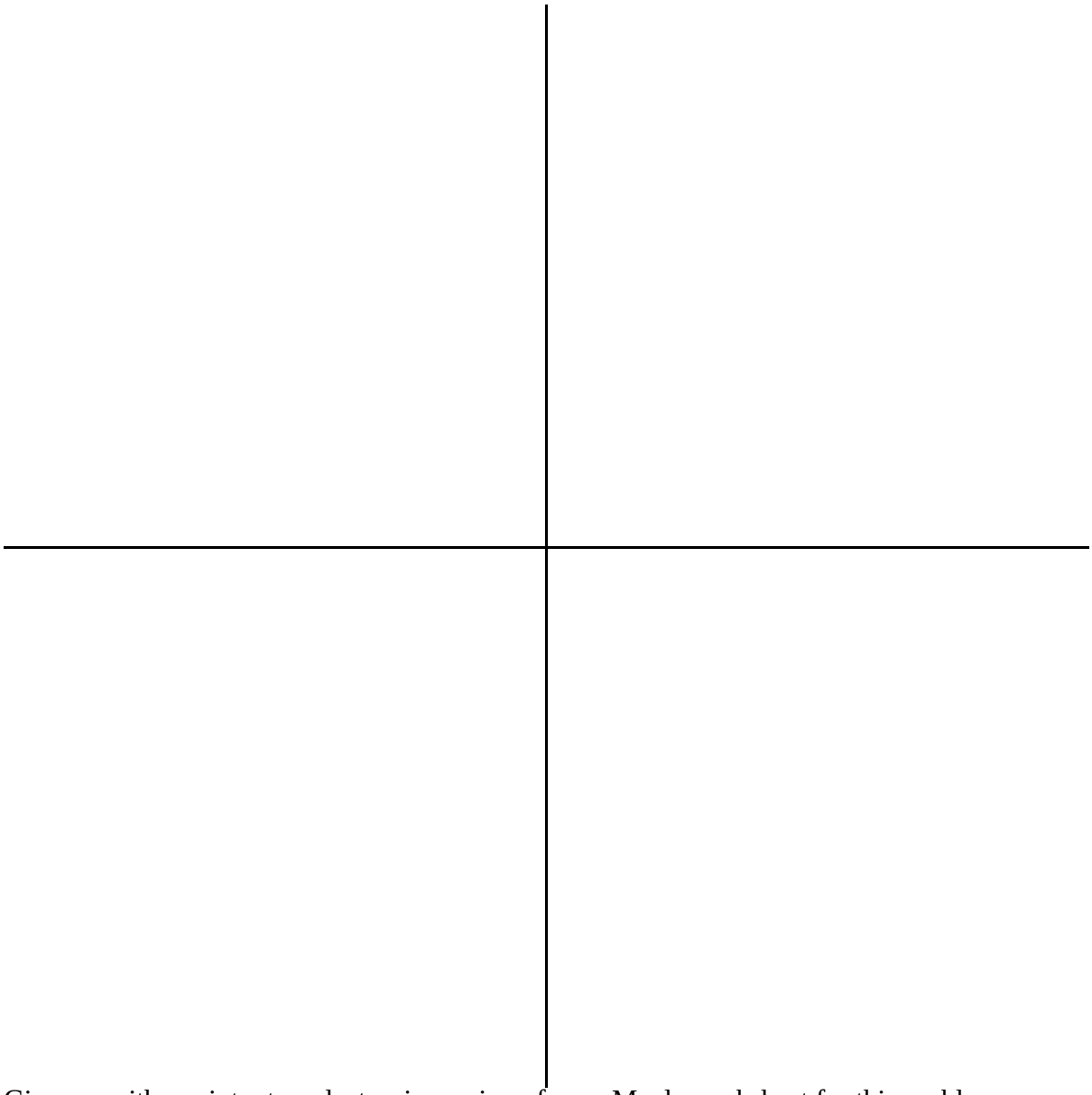
7. Show that the function given by f(x) = $\sqrt[3]{x}$ is continuous at x = 0 but is not differentiable at x = 0.

8. Find the equation of the tangent to the graph of f(x) = $\dfrac{1}{x}$ that intersects the x-axis at the point (5,0).

9. Suppose we know that f´(a) exists. Why can we say
$$\lim_{h \to 0} f(a+h) = f(a)?$$

10. Use Maple to get a semi-accurate graph of the following function. Be sure to give me information regarding the domain of the function, the location of x and y intercepts, vertical asymptotes and horizontal asymptotes.

$$g(x) = \frac{x^4 + x^3 - x^2 + x - 2}{x^4 - 2x^3 - 2x^2 + 2x + 1}$$



Give me with a printout or electronic version of your Maple worksheet for this problem.

Math 111 - Exam 1c

1. Compute the following limits. Give reasons for your work.

a) $\lim_{x\to 5} \dfrac{x^2 - 10x + 25}{x^2 - 5x}$ b) $\lim_{x\to 5} \dfrac{x^2 - 5x}{x^2 - 10x + 25}$

c) $\lim_{x\to 0} \dfrac{x}{\tan(3x)}$ d) $\lim_{x\to \infty} \dfrac{\sin x}{x}$

2. Use the definition of derivative to do the following.

a) Compute $f'(x)$ for the function $f(x) = \sqrt{x-1}$.

b) Prove that $\dfrac{d}{dx}[cf(x)] = c\dfrac{d}{dx}[f(x)]$.

3. Compute the derivative for each of the following functions.

a) $f(x) = x^{10} + 2x^8 - x^7 + 13$. b) $f(x) = x^2 \tan(x)$ c) $s(t) = \dfrac{3t\sin(t)}{t+1}$

4. A particle travels according to the law $s = t^2 - 8t$ where t is measured in seconds and s in meters. Find
a) the average velocity over the first 10 seconds,
b) the instantaneous velocity at 10 seconds,
c) the position of the particle when it is traveling –2 meters/ second.

5.Let $f(x) = \begin{cases} 2x & \text{for } x \le 0 \\ x^2 + 1 & \text{for } x > 0 \end{cases}$

a) Show that f is not continuous at x = 0.

<u>Note</u>: a good graph is worth some partial credit but is not a complete answer by itself.

b) Can f(x) be made continuous at x = 0 by defining f(0) suitably? Justify your answer.

6. In class we proved that $\dfrac{d}{dx}[x^n] = nx^{n-1}$. The method of this proof required that n be an integer and that n 1. Use this result to prove that the same is true for negative integers. That is, prove that $\dfrac{d}{dx}[x^{-n}] = -nx^{-n-1}$.

Hint: $x^{-n} = \dfrac{1}{x^n}$

Math 111 Exam1c
take-home portion

7. Find out all you can about the graph of the function

$$g(x) = \frac{2x^4 + x^3 - 8x^2 - 4x}{x^4 - x^3 - x^2 - x - 2}$$

You should submit a good graph and be sure the report of your investigation includes information on each of the following:

1) intercepts,

2) asymptotes (horizontal and vertical),

3) points at which the tangent line is horizontal,

4) any points of discontinuity.

Please show all work that supports your answers. Computations which justify finding exact points will receive more credit than approximations. For example, you will receive more credit if you use the Maple solve (and possibly diff) commands to help you find the exact points at which the tangent line is horizontal as opposed to approximating the values from graphs.

Math 111 - Exam 1d

1. What does it mean to say a function f is continuous at x = a?

2. For a function f, what is the definition of the derivative f´(x)?

3. Using the definition of derivative in problem 2, find k´(x) for k(x) = $x^2 - 7x$.

4. Using the definition of derivative in problem 2, find g´(x) for g(x) = $\sqrt{3x+4}$.

5. a) What is the domain of g(x) in problem 4?

b) What is the domain of the g´(x) found in problem 4?

6. For each set of conditions below sketch a graph which illustrates a function with that behavior.

a) f(x) is continuous at x = 0 but is not differentiable at x = 0.

b) $\lim\limits_{x\to 0} f(x)$ and f(0) both exist but are not equal

c) $\lim\limits_{x\to 0^+} f(x)$ and $\lim\limits_{x\to 0^-} f(x)$ both exist but $\lim\limits_{x\to 0} f(x)$ does not exist.

d) $\lim\limits_{x\to 0^+} f(x)$ exists but $\lim\limits_{x\to 0^-} f(x)$ does not exist.

7. Evaluate the following limits.

a) $\lim\limits_{x\to 0^+} 3^{1/x}$ b) $\lim\limits_{x\to 0^-} 3^{1/x}$

8. Locate the intercepts, asymptotes, and missing points of the following function and then sketch a graph showing these features.

$$f(x) = \frac{2x^3 - 8x}{x^3 - 8x^2 + 16x}$$

Take home portion

9. Consider the function defined by $f(x) = \begin{cases} x^3 \sin\left(\frac{1}{x}\right) & \text{for } x \quad 0 \\ 0 & \text{for } x = 0 \end{cases}$

a) Using the definition of the derivative, find f´(x). MAPLE is able to compute the very difficult limit.

b) Note that the f´(x) found above is not defined for x = 0. Find f´(0) using the definition of the derivative at x = 0.

c) Show that the derived function f´(x) is continuous at x = 0.