

Math 111 - Exam 2a

1) Take the derivatives of the following. DO NOT SIMPLIFY!

a) $y = (\sqrt{3} + \frac{1}{x})(\sin(2x) - x^{-3})$ b) $y = \frac{\sqrt{x^2 - 1}}{x^2 + 1}$ c) $y = \tan(\sec^2 x)$

2) Find the following derivatives

a) Find $\frac{dy}{dx}$ given that $\sqrt{x+y} = y^3$, and $x = 3, y = 1$.

b) A, b, and h are each functions of t satisfying, $A = \frac{1}{2}bh$. Find $\frac{db}{dt}$ when $A = 100, h = 10,$

$\frac{dA}{dt} = 2,$ and $\frac{dh}{dt} = 1,$ and $\frac{dh}{dt} = 1$.

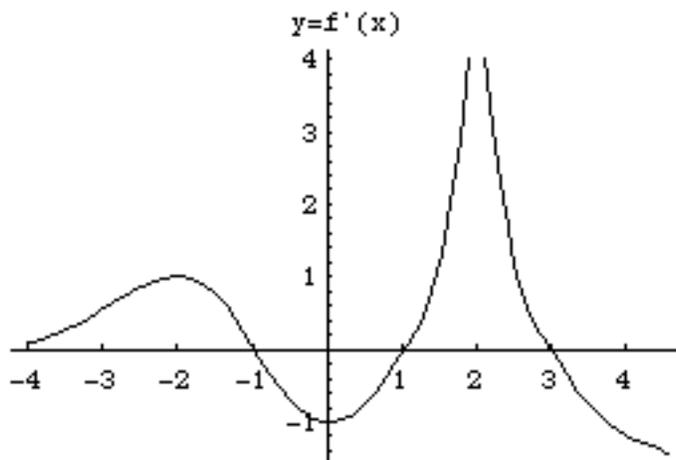
c) Find $f'(1)$, given that $f(x) = g(x^3 - x)$ and $g'(0) = -2$.

3) Let $f(x) = (x^2 + 4)^{1/3}$.

a) Show that $f(x) \cong 2 + \frac{1}{3}(x-2)$, for $x \cong 2$.

b) Does the estimate for $f(x)$ above, under-estimate, or over-estimate $f(x)$ for $x \cong 2$? Explain.

4) Below is the graph of the **derivative**, $f'(x)$, of a continuous function $f(x)$. From the graph of $f'(x)$ find the following about $f(x)$: the intervals of increasing and decreasing, the local minima and maxima, the intervals where f is concave up and concave down, the inflection points, and the graph of $f(x)$.



Math 111 - Exam 2b

1. State the Quotient Rule and give a simple, nontrivial example of its use.
2. Let $h(x) = \frac{f(x)}{g(x)} = f(x) (g(x))^{-1}$. Use this latter expression along with the Chain Rule and Product Rule to derive the quotient rule.
3. a) Find y' for $x^2 = \sqrt{y^3 + 2y^2}$ and give me the result in simplified form.
b) Find y'' for any (x,y) .
c) Find y' and y'' at the point $(-2,2)$. From these what can you say about the shape of the graph near this point?
4. The iteration scheme for Newton's Method is given by $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$.
What does Newton's Method give you and where does the formula above come from.
(You should be as detailed as possible, and you may want to use a picture to help describe your ideas.)
5. Sketch the graph of a "smooth" continuous function $y = f(x)$ having the following properties.
 - a) $f(-3) = -2, f(-2) = -1, f(-1) = 1, f(0) = 0, f(1) = -1$
 - b) $f'(x) < 0$ for $x < -3$ or $-1 < x < 1$ and $f'(x) > 0$ for $-3 < x < -1$ or $x > 1$
 - c) $f''(x) < 0$ for $-2 < x < 0$ and $f''(x) > 0$ for $x < -2$ or $x > 0$
6. a) State the Mean Value Theorem. b) Give two results mentioned in class that we can use the Mean Value Theorem to prove. c) Describe geometrically what the Mean Value Theorem says and give a graphical example of a function for which the Mean Value Theorem does not hold.
7. A rectangular plot of ground is to be enclosed by a fence and then divided down the middle by another fence. If the fence down the middle costs \$1 per yard and the perimeter fence cost \$2 per yard, find the dimensions of the plot of largest area that can be enclosed for \$480.
8. An automobile driver spots a child in the street 300 feet ahead. The car is travelling at 60 mph (88 ft/second). The car can decelerate at a rate of 13 ft/sec/ sec. Will the car stop in time to avoid hitting the child?
9. On the back graph the function given by $f(x) = \frac{x^3 + 3}{2x - 3x^2 - 2x^3}$. Label all vertical asymptotes, horizontal asymptotes, intercepts(x and y), critical points, and inflection points.

Math 111 - Exam 2c

1. a) Let $f(x) = 2x^3 - 7x^2 + 3x + 16$. Show that $f^{(IV)}(x)$, the 4th derivative of f with respect to x , is zero for all x . b) Let $s = 2t^3 - 9t^2$ be the equation of motion for a particle. Find the acceleration of the particle at each instant when velocity is 0.

2. Consider the graph of $x^3y = y^2 + 2x^3$.

- Find all points on the graph where the tangent to the graph is horizontal. (You must give x and y coordinates.)
- Determine the equation of the tangent line to the graph at the point $(2, 4)$.

3. The linear approximation (or linearization) of a function f at a point $x + Dx$ was given by the formula

$$f(x + Dx) \approx f(x) + f'(x) Dx.$$

- Explain in detail why this formula is useful and where it comes from. You need not do a derivation of the formula. (Hint: A picture would be a good idea though it is not sufficient.)
- Use linear approximation to approximate $\sqrt[3]{8.5}$.

4. Consider the function $f(x) = 3x^4 - 8x^3 + 6x^2 + 10$. Find all local extrema and all inflection points. For any local extremum, indicate whether it is a local maximum or a local minimum. Also determine the absolute maximum and minimum if they exist.

5. The profit, in dollars, earned from manufacturing x units of a particular product is given by the

formula $P(x) = 200 - 15x + 0.9x^2 - 0.01x^3$. Labor constraints dictate that at least 30 and no

more than 65 units are to be manufactured. How many units should be produced to earn the maximum profit?

6. a) Let $g(x) = x^{(2/3)} - 1$. Notice that although $g(-1) = g(1) = 0$, there is no point c in the interval $(-1, 1)$

where $g'(c) = 0$. Why doesn't this contradict Rolle's Theorem?

b) Use the Mean Value Theorem to prove that $-\pi < \sin(a) < \pi$ for all real numbers a .

Math 111-01, Calculus
Exam II, take-home portion

7. In the Newton's Method lab (part 4) you saw the following "classification" of what the initial point x_0 would produce for the function $f(x) = x^3 - x$:

$$x_0 < \frac{-1}{\sqrt{3}} \text{ and } \frac{1}{\sqrt{5}} < x_0 < \frac{1}{\sqrt{3}} \quad \text{Newton's Method converges to the zero } -1.$$

$$x_0 = \frac{\pm 1}{\sqrt{3}} \quad \text{Newton's Method fails (division by zero).}$$

$$x_0 = \frac{\pm 1}{\sqrt{5}} \quad \text{Newton's Method fails (period 2 point).}$$

$$\frac{-1}{\sqrt{5}} < x_0 < \frac{1}{\sqrt{5}} \quad \text{Newton's Method converges to the zero } 0.$$

$$\frac{-1}{\sqrt{3}} < x_0 < \frac{-1}{\sqrt{5}} \text{ and } x_0 > \frac{1}{\sqrt{3}} \quad \text{Newton's Method converges to the zero } 1.$$

Perform the same "classification" for the function $f(x) = x^3 - 2x$. That is, determine what Newton's Method will do for every possible initial point x_0 . Don't forget to justify all of your results.

Math 111 - Exam 2d

1. Compute the derivative of each of the following functions.

a) $f(x) = \sin^3 x \cos^2 x$ b) $f(x) = \tan(x^3 + 1)$ c) $f(x) = \frac{x+4}{x^2+3}$

2. Find y' for $x - x^2y = 4y^3$

b) Find the equation of the tangent to this curve at the point $(1, \frac{1}{2})$.

c) Find y'' in the above situation and determine which of the above points is a local maximum and which is a local minimum. Note that it is not necessary to simplify the formula for y'' .

3. a) State carefully the Mean Value Theorem including the hypotheses and the conclusion.

b) Use the Mean Value Theorem to show that if $f'(x) > 0$ for all x in an interval (a,b) , then $f(x)$ is increasing on that interval.

4. A fieldhouse has been proposed for The College of Wooster campus. The interior is to consist of a rectangular region with a semi circle at each end (see figure). If the perimeter of the room is to be a 200-meter running track, find the dimensions that will make the area of the rectangle as large as possible.

5. The acceleration of a rocket is given by $a(t) = t^{1/2} - t^2$. If it starts from rest on a platform 20 feet high, find the velocity function and the distance function.

6. An airplane is flying at an altitude of 6 miles and is on a course which will take it directly over a radar antenna (see figure). How fast is the plane flying, if the radar detects it at a range s of 20 miles with range closing (getting closer) at a rate of 300 miles per hour?

7. For the function $f(x) = 3x^5 + 5x^4 - 60x^3$, whose graph is shown below, find the intercepts, local extrema, and inflection points. (x - coordinates only)

Take home portion

8. Graph the function given below and label (both coordinates) all points of interest (intercepts, missing points, extrema, and inflection points) along with vertical and horizontal asymptotes.

$$f(x) = \frac{x^6 - 3x^5 - 4x^4 - x^3 + 3x^2 + 4x}{2x^6 - 13x^5 + 18x^4 + 15x^3 - 32x^2 + 28x - 48}$$

9. Using Newton's Method find to an accuracy of four decimal places the values of x for which the two functions $f(x) = \sin x$ and $g(x) = x^2 - 1$ intersect.