# Math 112 - Exam 1a

1. Calculate the derivative for each of the following functions:

a) $g(x) = \dfrac{2^x}{x}$ b) $f(x) = \sin^{-1}(1-3x)$ c) $h(x) = e^{x\cos x}$

2. Suppose that tests on a fossil show that 70% of its carbon-14 has decayed. Estimate the age of the fossil, assuming a half-life of 5750 years for carbon-14.

3. Evaluate the following limits:

a) $\lim e^{\frac{x}{3-x}}$ b) $\lim 2^{\frac{x}{1-x}}$ c) $\lim \dfrac{e^{-x}}{x^2}$ d) $\lim (e^x + x)^{\frac{1}{x}}$

4. Find the volume generated when the region R bounded by $y = e^{-x^2}$, the x-axis, the y-axis, and x=1 is revolved around the y-axis.

5. Let $g(x) = \dfrac{e^{-x}}{e^{-x}+1}$

a) Find lim g(x). b) Find lim g(x).

c) What is the range of g? d) Show that g is a one-to-one function.

e) Find $g^{-1}(x)$ . f) What is the domain of $g^{-1}$ ?

6. Show how the derivative of $y = \sec^{-1}(x)$ is derived (i.e. prove that the derivative has the form that it does).

7. Evaluate the following integrals:

a) $\displaystyle\int_e^{e^5} \frac{1}{x(\ln x)}\,dx$ b) $\displaystyle\int \tan^3 x \sec^5 x\,dx$ c) $\displaystyle\int \ln x\,dx$

d) $\displaystyle\int_4^8 \frac{\sqrt{x^2-16}}{x}\,dx$ e) $\displaystyle\int \frac{2\sqrt{x}}{1+x}\,dx$

# Math 112 - Exam 1b

1.Define ln(x) as we did in class.

2.List 4 properties of the function given by $f(x) = \ln(x)$.

3.Show that $\ln(xa) = \ln(x) + \ln(a)$ for all $a,x > 0$?

4.Where does the number 'e' come from?

5. Show why $\frac{d(e^x)}{dx} = e^x$.

6. For $a > 0$ define $a^x$ for any real number x as we did it in class.

7.List 4 properties of the function given by $g(x) = a^x$ where $a > 0$.

8.a) Explain the process we used to define arctan(x).

b) Explain the process we used to find $\frac{d}{dx}(\arctan(x))$.

c) Simplify the expression $\sin(2\arctan(x))$.

9. a) What assumptions were made about the quantity q(t) in deriving the formula $q(t) = q_0 e^{ct}$?

b) If 50 grams of uranium decays to 40 grams in 5 days, how long will it be until 98% of the 50 grams has decayed?

10.An art critic is viewing a tapestry mounted on a wall. The tapestry is3 meters tall and hangs so that the lower edge is 1 meter above the critics eye level. How far from the wall should the critic stand in order to get the best view (maximum viewing angle) of the tapestry?

11.Explain the difference in finding the derivatives of $\pi^x$, $x^\pi$, and $x^x$.

# Math 112 - Exam 1c

1. Find the derivative for each of the following:

a) $y = \ln(\cos x)$ b) $f(x) = 5^{3x} + (3x)^5$ c) $h(x) = \sqrt[x]{x^2 + 1}$

2. a) Define the natural logarithm function *ln* as it was defined in class.

b) State why $\frac{d}{dx}(\ln x) = \frac{1}{x}$.

3. a) Theorem 8 of Chapter 8 in your text states that $y = y_0 e^{kt}$ is a solution to the differential equation $\frac{dy}{dx} = ky$ such that $y(0) = y_0$. Prove this.

b) Use Theorem 8 to solve the following problem.

According to one of Kirchoff's rules for electrical circuits, $E = Ri + L\frac{di}{dt}$ where constants E, R, and L denote the electromotive force, the resistance, and the inductance, respectively, and i denotes the current at time t. If the electromotive force is terminated ($E = 0$) at time $t = 0$, and if the current is $i_0$ at the instant of removal, determine i as a function of t. (Note that R and L remain as undetermined constants.)

4. Evaluate the following integrals:

a) $\int xe^{1-x^2}\,dx$ b) $\int \sin^3 x \cos^3 x\,dx$ c) $\int_0^4 \frac{1}{16 + x^2}\,dx$

5. The inverse cosine function $\mathrm{Cos}^{-1}$ was defined to be the inverse of the function given by

$y = \cos x$, $0 \le x \le \pi$. Prove that $\frac{d}{dx}(\mathrm{Cos}^{-1}x) = \frac{-1}{\sqrt{1 - x^2}}$.

6. a) State the domain of sin h.

b) Prove that sin h is increasing on all its domain. Thus sin h is one-to-one and we can define an inverse $\sin h^{-1}$.

c) An interesting property of $\sin h^{-1}$ is $\sin h^{-1} = \ln(x + \sqrt{x^2 + 1})$. Prove this.

# Math 112 - Exam 1d

1. Calculate the derivative for each of the following functions.

a) $f(x) = \ln(8x + x^4)$ b) $f(x) = \arcsin(3x)$ c) $f(x) = \mathrm{Tan}^{-1}(x+2)$

d) $f(x) = e^{\cos x} + 7^x$ e) $f(x) = (x+1)^{\tan x}$

2. The rate of growth of a population of sparrows is assumed to be proportional to the population size. If the population was estimated to be 2 million in 1970 and 2.4 million in 1976, what would the population be in 1991?

3. Integrate each of the following integrals. a) $\int x(4+x^2)^{3/2}\,dx$ b) $\int \frac{5\,dx}{1+x^2}$

c) $\int \sec x\,dx + \int \cot x\,dx$ d) $\int \sin^{12} x \cos^3 x\,dx$

4. For the function $g(x) = x^2 e^{-x^2}$, <u>find</u> and <u>identify</u> all the local extrema and then <u>sketch a graph</u> of g(x).

5. Find the volume generated when the area under the curve $y = 5e^{3x}$ on the interval [-3 0] is rotated about the x-axis.

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6. Evaluate the following integral. $\int x^{19} \sqrt{x^2 + 3}\, dx$.

7. Calculate the area enclosed by $y = \ln x$ and $y = (x - 3)^2$. Your answer should be expressed
as a decimal with at least three digits to the right of the decimal point.