Math 112 - Exam 2a

1. Evaluate the following limits. a) $\lim_{x\to\infty} \frac{(\ln(x))^2}{x^{1/7}}$ b) $\lim_{x\to 0} (1-5x)^{2/x}$

2. Use integration by parts on $\int x^n \sin ax \, dx$ to establish a reduction formula for this type of integral.

3. Integrate each of the following integrals.

a) $\int_1^\infty \frac{7dx}{(3x+5)^2}$ b) $\int_0^\infty \frac{7x\,dx}{5x^2+3}$ c) $\int \frac{7\,dx}{(x^2+9)^{3/2}}$

d) $\int x \sqrt[3]{x+2}\, dx$ e) $\int_{-3}^{0} \frac{dx}{\sqrt{9-x^2}}$ f) $\int \frac{7\,dx}{x^2+x}$ g) $\int 7x^2 e^x dx$

4. MAPLE cannot do the following integral. What substitution would you suggest to change the form of this integral so that MAPLE can do it ? $\int \frac{x^{1/2}dx}{x^{1/3}-x^{1/5}}$

Take Home Portion of Exam

5. $\int_0^{1/2} \frac{x^{1/2}dx}{x^{1/3}-x^{1/5}}$

6. $\int_0^\infty \frac{dx}{x^{2/3}+x^2}$

Math 112 - Exam 2b

1. Evaluate the following integrals:

a) $\int xe^{3x}dx$ b) $\int \frac{x^2}{1+x^2}\,dx$ c) $\int_1^{\infty} \frac{1}{x}dx$ d) $\int_0^3 \frac{dx}{(x-1)^{2/3}}$

2. Compute the following limits (Justify your answers.):

a) $\lim_{x\to\infty} \frac{e^x}{x^2}$ b) $\lim_{x\to 0} x^x$ c) $\lim_{x\to\infty} \sin \pi x$ d) $\lim_{n\to\infty} \sin \pi n$

3. What does it mean to say the infinite series $\sum_{k=1}^{\infty} a_k$ converges?

4. a) $\lim_{x\to\infty} \frac{x^2-3}{2x^2+3x+1}$

b) Does the series $\sum_{k=1}^{\infty} \frac{k^2-3}{2k^2+3k+1}$ converge or diverge? Why?

5. Recall the theorem: If the series $\sum b_k$ converges then the series $\sum cb_k$ also converges. (In fact, if the first series converges to L, then the second converges to cL.)

a) Give a specific example which proves that the following is not true: If the series $\sum ca_k$ converges then the series $\sum a_k$ also converges.

b) Prove that the statement in part a) is indeed true if we add the assumption that $c \neq 0$.

That is:
If the series $\sum ca_k$ converges and $c \neq 0$, then the series $\sum a_k$ also converges.

Math 112 Take-home Problem

6. Analyze

a) the infinite sequence $\lim_{k\to\infty} \frac{10^k - k^{10}}{k!}$

and

b) the infinite series $\sum_{k=1}^{\infty} \frac{10^k - k^{10}}{k!}$

Math 112 - Exam 2c

1.a) Show how one would use the technique of integration by parts to find an antiderivative by using it for a simple, nontrivial example.

b) Show how one would use the technique of trigonometric substitution to find an antiderivative by using it for a simple, nontrivial example.

c) Show how one would use the technique of partial fraction decomposition to find an antiderivative by using it for a simple nontrivial example.

d) Show how one would use the technique of simple substitution to find an antiderivative by using a simple, nontrivial example.

2.In using trigonometric substitution there are three possible substitutions one can make. What are they and in what situation would you use each particular substitution?

3.The technique of trigonometric substitution can be used in combination with completing the square. Illustrate this by using a simple, nontrivial example.

4.State l'Hôpital's rule and give a simple, nontrivial example of its use.

5. List as many of the indeterminate forms as you can remember and give an example of each one (don't compute the limits). For those forms which do not fit l'Hôpital's immediately indicate the technique for finding the limit by doing your example. If several indeterminate forms use the same technique, tell me and only use the technique on one of your examples.

6. Give examples of three different types of improper integrals and illustrate how one works with these by doing the integration.

7. What is the definition of a sequence?

8. Show that $\lim_{n\to\infty} (r^n) = 0$ if $|r| < 1$ by using the meaning we assigned to limit.

Math 112 - Exam 2d

1. Determine whether each integral converges or diverges. If it converges, find the value of the integral.

a) $\displaystyle\int_0^{\infty} \frac{e^x}{1+e^{2x}}\,dx$ b) $\displaystyle\int_0^{e} \frac{\ln x}{x}\,dx$

2. Give an example of each of the following, showing all reasoning. If no example exists, write "impossible".

a) A geometric series which converges to 1. b) An oscillating sequence which converges to 1.

c) A nonconstant sequence which converges to 1. d) A monotone sequence which diverges.

e) A convergent series $\displaystyle\sum_{k=1}^{\infty} a_k$ such that $\displaystyle\lim_{k\to\infty} a_k = 1$

3. Find the arc length of the curve $x = \frac{1}{3}\left(y^2+2\right)^{3/2}$ from y = 0 to y = 1.

4. Determine whether the following series converge or diverge. For any that converge, find the sum.

a) $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^2+3k+2}$ b) $\displaystyle\sum_{k=1}^{\infty} \frac{1}{4+2^{-k}}$ c) $\displaystyle\sum_{k=1}^{\infty} \frac{5^{k+1}}{e^{2k}}$ d) $\displaystyle\sum_{n=1}^{\infty} \ln\left(\frac{n}{n+1}\right)$

5. Let R be the region which is bounded by $y = 2x - x^2$ and the x-axis. Find the y-coordinate ONLY ($\bar{y}$) of the centroid of the region R.

6. Determine whether the sequence converges or diverges. If it converges, state the limit.

a) $\left\{\dfrac{e^{n^2}}{n^2}\right\}_{n=1}^{\infty}$ b) $\left\{\arctan\left(\dfrac{2-e^{4n}}{n}\right)\right\}_{n=1}^{\infty}$