Math 112 Exam 3a

1.Determine whether the following series converge absolutely, converge conditionally, or diverge.

a) $$\sum_{1}^{\infty} (-1)^n \frac{3n^2 - 5n + 7}{2n^4 + 4n^2 + 8}$$

b) $$\sum_{1}^{\infty} (-1)^n \sin\left(\frac{\pi n}{3}\right)$$

c) $$\sum_{2}^{\infty} (-1)^n \frac{\ln n}{n^{3/2}}$$

d) $$\sum_{1}^{\infty} (-1)^n \frac{3n + 1}{n^2 + 1}$$

2. Find the <u>interval</u> of convergence for each power series.

a) $$\sum_{0}^{\infty} \frac{n^2 x^n}{3^n}$$ b) $$\sum_{0}^{\infty} \frac{(-1)^n 2^n x^n}{5^n (n+1)}$$ c) $$\sum_{0}^{\infty} \frac{n! x^n}{(2n)!}$$

3.a) Give an example of a series $\sum a_n$ which converges but for which $\sum a_n^2$ diverges.

b) Give an example of a series $\sum a_n$ which diverges but for which $\sum a_n^2$ converges.

c) Using one of our tests show that if $\sum a_n$ is a <u>positive</u> series which converges, then $\sum a_n^2$ must also converge.

4. a) Define <u>carefully</u> and <u>precisely</u> the meaning of "$\sum_{0}^{\infty} a_n = S$", that is of "the series $\sum_{0}^{\infty} a_n$ converges to S".

b) Given an example of a <u>bounded sequence</u> which has no limit.

Math 112 Exam 3b

1.What does it mean to say the infinite series $\sum_{k=1}^{\infty} a_k$ converges? Be sure to define all terms or notation you use.

2.Consider the infinite series given by $\sum_{k=1}^{\infty} \frac{1}{k(k+1)}$.

a) Find a formula for its sequence of partial sums.

b) Using this show that the series converges and find what it converges to.

3.If we have a series $\sum_{k=1}^{\infty} a_k$ for which $a_k \geq 0$, why is it sufficient to show that the sequence of partial sums is bounded above in order to know that the series converges?

4.What does it mean to say the series $\sum_{n=1}^{\infty} a_n$ converges absolutely? What does it mean to say this series converges conditionally?

5.Give an example of a series that converges conditionally.

6. a) Find an upper bound for the sequence of partial sums of the positive term series $\sum_{n=1}^{\infty} \frac{1}{n^3+n}$.

b) Show that the sequence of partial sums of the positive term series given by $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n+1}}$ is unbounded.

7.Determine for what values of x the power series $\sum_{n=0}^{\infty} \frac{(2x-1)^n}{\ln(n+2)}$ converges.

8.a) Find the Taylor series for $f(x) = 2^x$ at $c = 1$.

b) For what values of x is 2^x equal to its Taylor series. (i.e. for what values of x is $\lim_{n\to\infty} R_n(x) = 0$?

9. What can you tell me about the equation $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$?

10. Use a suitable rotation of axes to find an equation for the graph of

$$3x^2 - 2\sqrt{3}xy + y^2 - (2 + 4\sqrt{3})x + (4 - 2\sqrt{3})y + 4 = 0$$

in an x´y´ coordinate system. What kind of object is the graph? Give the xy coordinates of any vertices?

Recall: $x = x'\cos(q) - y'\sin(q)$ and $y = x'\sin(q) + y'\cos(q)$

Math 112 Exam 3c

1. Determine whether or not the given series converge.

a) $\displaystyle\sum_{k=1}^{\infty} \frac{\sqrt{k}}{2^k}$ b) $\displaystyle\sum_{k=2}^{\infty} \frac{1}{k\ln(k)}$ c) $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^k}{\sqrt{k}}$

2. a) What does it mean for a power series to converge absolutely?

b) Consider the series $\displaystyle\sum_{k=2}^{\infty} \frac{(-3)^k x^k}{\ln(k)}$. Determine the interval of convergence, identifying points where the convergence is absolute and points where the convergence is conditional. Justify your answers.

3. Consider the equation $Ax^2 + Cy^2 + Dx + Ey + F = 0$. Tell me all you can about the graph of this equation in each of the following cases. (For example if $A = C = 0$, $D > 0$ and $E > 0$ the graph will be a line with positive slope.)

a) $A = 0, C > 0, D > 0$ b) $A = C$ c) $A > 0, C < 0$

4. Graph $x^2 - 6x + 2y^2 - 8y + 8 = 0$. Label all important points (intercepts, foci, vertices, center) and features of the graph.

5.a) Show that the Taylor Series for $f(x) = \dfrac{1}{1-x}$ about the point $x = 0$ is

$1 + x + x^2 + x^3 + \ldots + x^n + \ldots$.

Similarly one can show that the Taylor Series for $g(x) = \dfrac{1}{1-\frac{1}{x}}$ about the point $x = 0$ is

$$1 + \frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3} + \ldots + \frac{1}{x^n} + \ldots .$$

b) Now, the following computation uses the results above: For $x > 0$

$$2 < (1 + x + x^2 + x^3 + \ldots + x^n + \ldots) + \left(1 + \frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3} + \ldots + \frac{1}{x^n} + \ldots\right)$$

(since the right hand side is all positive terms and includes a 1 + 1)

$$= \frac{1}{1-x} + \frac{1}{1-\frac{1}{x}} = \frac{1}{1-x} - \frac{x}{1-x} = \frac{1-x}{1-x} = 1$$ What is wrong!

Bonus: Determine whether or not the following series converges. Justify your answer.

$$\sum_{k=2}^{\infty} \frac{2^k}{2^{2k} - 4}$$

Math 112 Take-home Problem

6. a) Determine the Taylor Polynomials of degrees 0, 1, 2, and 3 for $f(x) = \ln(1+x)$ about the point $x = 0$.

b) Graph them on a coordinate axis with f.

c) From the *graph* of $P_3(x)$ and $\ln(1+x)$, estimate the error in evaluating $\ln(2)$.

d) Determine a bound on the accuracy of the approximation $\ln(2) \approx P_3(2)$. Compare this with your estimate in part c).

e) Determine how large n must be to ensure accuracy to four decimals in the approximation $\ln(2) \approx p_n(2)$.

Math 112 Exam 3d

1. Determine whether the following series converge absolutely, converge conditionally, or diverge.

a) $\displaystyle\sum_{n=1}^{\infty} \frac{7^n - 4}{2 + 3^n}$ b) $\displaystyle\sum_{k=2}^{\infty} (-1)^k \frac{1}{k \ln k}$ c) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \cos\left(\frac{1}{n}\right)$

d) $\displaystyle\sum_{k=1}^{\infty} \left(\frac{k}{k+1}\right)^{k^2}$ e) $\displaystyle\sum_{k=1}^{\infty} \frac{4 - \tan^{-1} k}{k^3}$

2. Find the radius of convergence of the following power series: $\displaystyle\sum_{n=1}^{\infty} \frac{(nx)^n}{n!}$

3. Find the interval of convergence for the following power series:

a) $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^k (x+5)^k}{3^k \ln k}$ b) $\displaystyle\sum_{k=1}^{\infty} \frac{(9)^k k! x^k}{(2k)!}$

4. a) Find the 4th Taylor polynomial, $p_4(x)$, for $f(x) = \ln(x-1)$ about $a = 2$.

b) Find Lagrange's form of the remainder $[R_4(x)]$ associated with $p_4(x)$.

c) State the relationship between $p_4(x)$, $R_4(x)$ and $f(x)$.

5. Use any method to find the Maclaurin series for the following functions.
Give the interval over which this representation is valid.

a) $f(x) = \dfrac{x}{1 + 9x^2}$ b) $g(x) = \cos(x^2)$